



## Book of Abstracts

**ICSMM'2025**

***International Congress on Statistics and Mathematical Modeling***

October 21–22, 2025

Faculty of Sciences, Chouaib Doukkali University in El Jadida, Morocco

Organized by the Team of Statistics and Mathematical Modeling (ESMM)  
in Faculty of Sciences, El Jadida

### Contact

Pr. Echarif EL HARFAOUI (Chair) – [elharfaoui.e@ucd.ac.ma](mailto:elharfaoui.e@ucd.ac.ma)

Pr. Omar BALATIF (Co-chair) – [balatif.maths@gmail.com](mailto:balatif.maths@gmail.com)

Pr. Otmane BENCHIHEB (Co-chair) – [otmane.benchiheb@gmail.com](mailto:otmane.benchiheb@gmail.com)

Site web: <https://icsmm25.sciencesconf.org/>

## Introduction and Objectives

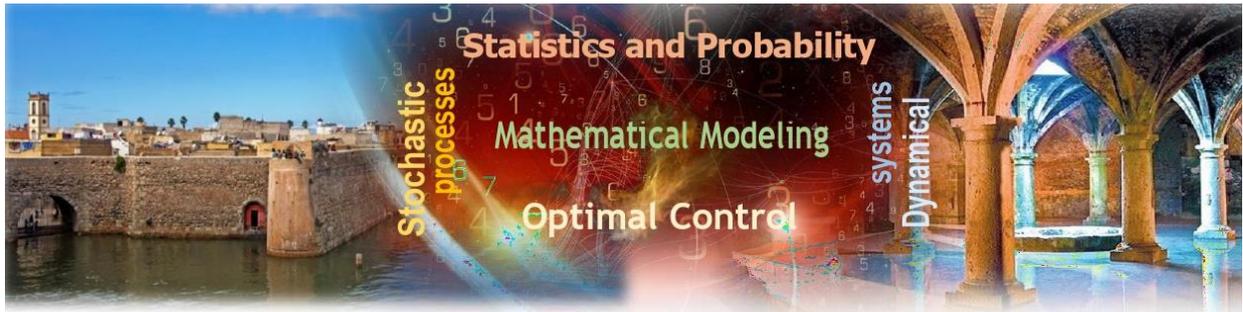
This international congress is part of the meetings between academics, scientists and experts in mathematics and its applications to focus on advances, new methodologies and new results of theoretical and practical research in disciplines related to dynamic systems:

- Statistics and Probability    - Stochastic processes
- Mathematical modeling    - Optimal control of dynamical systems
- Mathematical methods in Dynamical systems

## Sessions & Topics

*ICSMM'2025* features three sessions, each covering, but not limited to, a diverse range of research areas as follows:

- ✓ **Statistics, Probability and Stochastic processes**
  - Mathematical Statistics
  - Statistical data
  - Time series data
  - Stochastic processes
  
- ✓ **Mathematical modeling and Optimal control of dynamical systems**
  - Stochastic modeling
  - Mathematical modeling
  - Optimal control
  
- ✓ **Mathematical methods in Dynamical systems**
  - Numerical methods
  - Mathematical system



The Team of Statistics and Mathematical Modeling in Faculty of Sciences, Chouaib Doukkali University organizes

## 1st International congress on Statistics and Mathematical Modeling

October 21-22, 2025 El Jadida – Morocco

This international congress is part of the meetings between academics, scientists and experts in mathematics and its applications to focus on advances, new methodologies and new results of theoretical and practical research in disciplines related to dynamic systems:

- Statistics and Probability - Stochastic processes
- Mathematical modeling - Optimal control of dynamical systems
- Mathematical methods in Dynamical systems

### Invited Speakers

- |              |  |                      |   |
|--------------|--|----------------------|---|
| ▪ M. Elhia   | FSJES, UH2, Casablanca, Morocco        | ▪ J. L. Da Silva     | University of Madeira, Portugal             |
| ▪ G. Mélard  | Université Libre de Bruxelles, Belgium | ▪ J. Ngatchou-Wandji | IECL, Nancy, University of Lorraine, France |
| ▪ I. Ouassou | ENSA, UCA, Marrakech, Morocco          | ▪ M. Rachdi          | FSHS, University of Grenoble Alps, France   |

### Scientific Committee

- |                      |  |
|----------------------|--|
| ▪ S. Amine           | FST, UH2, Mohammedia, Morocco          |
| ▪ O. Balatif         | FS, UCD, El Jadida, Morocco            |
| ▪ O. Benchiheb       | FS, UCD, El Jadida, Morocco            |
| ▪ J. L. Da Silva     | University of Madeira, Portugal        |
| ▪ E. El Harfaoui     | FS, UCD, El Jadida, Morocco            |
| ▪ M. Elhia           | FSJES, UH2, Casablanca, Morocco        |
| ▪ M. Erraoui         | FS, UCD, El Jadida, Morocco            |
| ▪ M. Harel           | ESEP, University of Limoges, France    |
| ▪ A. Kaddar          | ENSA, UCD, El Jadida, Morocco          |
| ▪ G. Mélard          | Université Libre de Bruxelles, Belgium |
| ▪ J. Ngatchou-Wandji | IECL, University of Lorraine, France   |
| ▪ I. Ouassou         | ENSA, UCA, Marrakech, Morocco          |
| ▪ M. Rachdi          | FSHS, Université de Grenoble, France   |

### Organizing Committee

- |                  |                                |
|------------------|--------------------------------|
| ▪ M. Alahiane    | ENCG, UCD, El Jadida, Morocco  |
| ▪ M.S.E. Arrouch | FS, UCD, El Jadida, Morocco    |
| ▪ N. Asrir       | FSS, UCA, Marrakech, Morocco   |
| ▪ Z. Bahraoui    | ESTSB, UCD, El Jadida, Morocco |
| ▪ O. Balatif     | FS, UCD, El Jadida, Morocco    |
| ▪ O. Benchiheb   | FS, UCD, El Jadida, Morocco    |
| ▪ E. El Harfaoui | FS, UCD, El Jadida, Morocco    |
| ▪ K. El Kimakh   | FS, UCD, El Jadida, Morocco    |
| ▪ M. El Omari    | FPSB, UCD, El Jadida, Morocco  |
| ▪ Z. El Rhoubari | ESTSB, UCD, El Jadida, Morocco |
| ▪ M. Erraoui     | FS, UCD, El Jadida, Morocco    |
| ▪ J. Lahrache    | FS, UCD, El Jadida, Morocco    |
| ▪ N. Ziane       | FS, UCD, El Jadida, Morocco    |

### Coordinator:

Echarif Elharfaoui

Department of Mathematics, Faculty of Science El Jadida,  
Chouaib Doukkali University

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[icsmm25@sciencesconf.org](mailto:icsmm25@sciencesconf.org)

Event website: <https://icsmm25.sciencesconf.org>

### Important dates

- Registration deadline: September 05, 2025
- Abstract submission deadline: September 10, 2025
- Notification of acceptance: October 15, 2025





## Scientific Program

Tuesday, 21th October 2025

09:00 – 10:30	<b>Opening Ceremony and Welcome Reception</b>
10:30 – 11:00	<b>Coffee break</b>
11:30 – 12:30	<p><b>Plenary Conference:</b> Adaptive ridge estimator in a linear regression model with spherically symmetric errors under constraint</p> <p style="text-align: center;">Pr Idir OUASSOU (ENSA, Université Cadi Ayyad, Marrakech, Maroc)</p> <p>Chair: Pr Echarif EL HARFAOUI</p>
12:30 – 13:30	<p><b>Plenary Conference:</b> Time-dependent ARMA and VARMA models, and application to seasonal adjustment</p> <p style="text-align: center;">Pr. Guy MELARD (ECARES, Université Libre de Bruxelles, Belgique)</p> <p>Chair: Pr Echarif EL HARFAOUI</p>
13:30 – 15:00	<b>Lunch break</b>

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	<b>Session 1: Stochastic Modeling</b>		<b>Session 2: Probability and Statistics</b>		<b>Session 3: Stochastic Process</b>	
	<ul style="list-style-type: none"> <li>▪ Salle: 1</li> <li>▪ Chairs: Pr Z. Bahraoui</li> </ul>		<ul style="list-style-type: none"> <li>▪ Salle: 2</li> <li>▪ Chairs: Pr G. Méléard</li> </ul>		<ul style="list-style-type: none"> <li>▪ Salle: 3</li> <li>▪ Chairs: Pr J. Ngatchou-Wandji</li> </ul>	
15:00 - 15:15	Pr Elmehdi FARAH	Markovian-Switched SIR Epidemic Model with Two Coexisting Strains	Pr Ayad EL BAZ	Mathematical Foundations of Vector Autoregressive Modeling and an Application to Macroeconomic Data	Pr Mohamed EL OMARI	Parameter estimation of a tempered fractional Brownian motion with drift
15:15 - 15:30	Mohamed EL YAHYAOUI	Deterministic and Stochastic Analysis of an Integrated Economic Model: Growth and Indicators Simulation	Pr Lahoucine HOBBAD	Advanced Shrinkage Estimation for Symmetric Models	Abdelkarim OUALAID	A mild approach to spatial discretization for backward stochastic differential equations in infinite dimensions
15:30 - 15:45	Noureddine OULDKHOUIA	Towards a Probabilistic Theory of Deep Learning	Mohamed Salah Eddine ARROUCH	A Generalized Test for Volatility Regime Shifts in Parametric Time Series	Sara SOULAIMANI	Analyse stochastique et stabilité globale d'un modèle de consommation de drogues avec étude de sensibilité statistique
15:45 - 16:00	Noureddine OULDKHOUIA	From Robbins-Monro to Adam: Stochastic Approximation Methods in Deep Learning Optimization	Mohamed Amine ELAAFANI	Detecting Structural Change in Autoregressive Model Using Likelihood Ratio Test	Khadija BENCHEIKH	Heroin Use Dynamics with Relapse: A Stochastic Model with General Nonlinear Incidence
16:00 - 16:15	Jaouad IGBIDA	Mathematization of Competency Mastery Processes: Study of Three Effort Configurations	Sara NEJJAM	Rupture detection methods	Hassan GOUTI	Portfolio Optimization: Dynamic Programming Methods
16:15 - 16:30	Jaouad IGBIDA	Modeling Intervention Strategies for Information Campaigns and Exact Information vs. Error Containment in Dynamic Social Networks	Pr Echarif EL HARFAOUI	Testing for multiple change-points for absolutely regular observations	Aziza LAOUINA	Stationary Distribution and Extinction in a Stochastic Model of Multidrug-Resistant Tuberculosis
16:30 - 16:45	Meryiem EL FARRAGUI	Analytical Solution of Oscillatory Newtonian Plane Couette Flow and Stokes' Second Problem with Dynamic Wall Slip	Abdelati LAGZINI	Estimating R0 in Partially Observed Stochastic SIR Models: Bayesian and Classical Approaches	Pr Omar BALATIF	Optimal control a spatio-temporal problem
16:45 – 17:00	<b>Coffee break</b>					

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**Wednesday, 22th October 2025**

<b>Session 1: Mathematical Modeling and Optimal control (1)</b> ▪ Salle: 1 ▪ Chairs: Pr O. Balatif		<b>Session 2: Statistics</b> ▪ Salle: 2 ▪ Chairs: Pr I. Ouassou		<b>Session 3: Mathematical Modeling and Optimal control (2)</b> ▪ Salle: 3 ▪ Chairs: Pr A. Kaddar		
09:00 - 09:15	Sanaa EL FADILY	Stability Analysis of a Delay Differential Model for Unemployment Dynamics	Pr Mohamed ALAHIANE	Functional Projection Pursuit Regression within a Generalized Partially Linear Single-Index Framework	Tawfik JABER	Gradient Optimal Control of an Infinite Dimensional Bilinear System Using Bounded Distributed Controls
09:15 - 09:30	Meriem ELKAF	Fractional Mathematical Analysis of Tobacco Consumption	Pr Mounir BOUMAHDI	Estimation in nonparametric functional-on-functional models with surrogate responses	Mohamed BAROUDI	A coinfection Model of Bacterial Meningitis Diseases with Optimal Control
09:30 - 09:45	Anas BENHACHEM	The Impact of Pricing Strategies and Advertising on Market Shares of two competing firms: A Differential Game Model Approach in an Oligopoly	Nadia ASRIR	Dimension Reduction and Variable Selection for High-Dimensional Right-Censored Data	Oumaima AARABATE	Optimal control analysis of an unemployment model with an online freelancing intervention
09:45 - 10:00	Mourad OUYADRI	Mathematical Model of the Monkeypox Epidemic with Waning Immunity and Staged Infectiousness	Ismail ARJDAL	High-Dimensional Sparse Clustering for Bank Marketing Data	Samira ZOUHRI	Stochastic Optimal Control of Epidemic Transmission
10:00 - 10:15	Salaheddine BELHDID	A Mathematical Framework for Fake Rumor Dissemination	Slimane REGUI	Detecting error correlation in a periodic multiple Regression model	Mohcine EL BAROUDI	Optimal Control of Epidemics with Time Delay
10:15 - 10:30	Imane SMOUNI	Spatiotemporal Stability Analysis of Soil-Borne Disease Dynamics in Tomato Plants	Jilali STOUTOU	Non parametric estimation by B-spline approach for the Gaussian Vasicek-type processes with random effects	Amina ESSAMY	Optimal Control Strategies for Blood Glucose Regulation in Type 1 Diabetes
10:30 - 10:45	Jaouad EL MESKAOUI	A Mathematical Modeling and Analytical Approach to Controlling the Spread of Errors and Learning in Social Networks			Pr Abdellilah KADDAR	Economic Modeling Using Mixed Differential Equations to Anticipate the Unexpected
10:45 - 11:00	<b>Coffee break</b>					

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11:00 - 12:00	<b>Plenary Conference: Systèmes dynamiques en contexte socio-économique : stabilité et optimisation</b> Pr Mohamed EL HIA (FSJES, Ain Sebaa Université Hassan 2 de Casablanca, Maroc) Chair: Pr Omar BALATIF					
12:00 - 13:00	<b>Plenary Conference: Testing for multiple change-points</b> Pr Joseph NGTCHOU-WANDJI (IÉCL, Université de Lorraine, Nancy, France) Chair: Pr Echarif EL HARFAOUI					
13:30 - 15:00	<b>Lunch break</b>					
	<b>Session 1: Mathematical methods in Dynamical systems</b> ▪ Salle: 1 ▪ Chairs: Pr O. Benchiheb			<b>Session 2: Mathematical Modeling and Optimal control (3)</b> ▪ Salle: 2 ▪ Chairs: Pr M. Elhia		
15:00 - 15:15	Mohammed EL BILALI	On the properties of the gDMP inverse of an operator	Sofiane KHASSAL	Mathematical Modeling, Analysis, and Optimal Control of the cochineal insect impact on cacti plants		
15:15 - 15:30	Abdellah ELHYAT	Titchmarsh's theorem for the quaternion linear canonical transform	Khadija OUBOUSKOUR	Cost-Effectiveness Analysis of Control Strategies for Paratyphoid Transmission		
15:30 - 15:45	Said CHABLAOUI	The quantum Adams-Bashforth-Moulton fractional method for solving quantum fractional differential equations	El Mehdi MOUMINE	Modeling and Analysis of a Fractional order Spatio-temporal SEIR Model: Stability and Prediction		
15:45 - 16:00	Abbes KADYRI	Adaptive Spline Interpolation Approach for Global MPPT in Photovoltaic Systems	Habib HASSOUNI	Mathematical Analysis and Control of Merchant Behavior in E-Commerce Systems		
16:00 - 16:15	Ayoub MOUNIR	On skew generalized quasi-cyclic codes	Kawtar LARRACHEE	Contribution of vaccination coverage in the fight against epidemics: A mathematical vaccino-temporal SEIRS model		
16:15 - 16:30	Mohamed Amine IGHACHANE	Jensen's Inequality for $M_\psi$ -Convex Functions and Operator Versions with Applications	Khadija OUBOUSKOUR	Optimal Control of Cutaneous Leishmaniasis Dynamics and Reservoir Limitation		
16:30 - 16:45	M'hamed MABROUK	On the relations between different dual bilevel problems	Jamal BALLAHI	Modélisation de la Fièvre Aphteuse dans un Contexte d'Immigration Active		
16:45 - 17:00	Ahmed ED-DAHDAH	Approximate strong and weak subdifferentials of the difference of two set-valued mapping	Salma MOUJID	Stability Analysis of an Unemployment Model with Nonlinear Job Creation Dynamics		
17:00 - 17:30	<b>Coffee break</b>					
17:30 - 18:00	<b>Closing Ceremony</b>					

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## Participants ICSMM'2025

Last Name	First Name	title/Subject	Institution	Co-authors
<b>Topics: Statistics, Probability, Stochastic processes</b>				
Pr ALAHIANE	Mohamed	Functional Projection Pursuit Regression within a Generalized Partially Linear Single-Index Framework	ENCG, El Jadida, Chouaib Doukkali University	I. Ouassou (ENSA, Marrakech) and M. Rachdi (Grenoble Alps University)
Pr BOUMAHDJ	Mounir	Estimation in nonparametric functional-on-functional models with surrogate responses	FS, Ain Chock, Casablanca, Hassan 2 University	I. Ouassou (ENSA, Marrakech) and M. Rachdi (Grenoble Alps University)
ASRIR	Nadia	Dimension Reduction and Variable Selection for High-Dimensional Right-Censored Data	FSS, Marrakech, Cadi Ayyad University	A. Mkhadri (FSS, Marrakech) and T. Bouezmarni (Sherbrooke University)
ARJDAL	Ismail	High-Dimensional Sparse Clustering for Bank Marketing Data	FS, El Jadida, Chouaib Doukkali University	E. Elharfaoui (FS, El Jadida) and M. Alahiane (ENCG, El Jadida)
REGUI	Slimane	Detecting error correlation in a periodic multiple regression Model	FST, Tanger, Abdelmalek Essaadi University	A. Akharif (FST, Tanger) and A. Mellouk (CREF, Tanger)
STOUTOU	Jilali	Non parametric estimation by B-spline approach for the Gaussian Vasicek-type processes with random effects	FST, Béni Mellal, Sultan Moulay Slimane University	H. El Maroufy and H. Chaouch (FST, Béni-Mellal)
Pr EL BAZ	Ayad	Mathematical Foundations of Vector Autoregressive Modeling and an Application to Macroeconomic Data	ENCG, El Jadida, Chouaib Doukkali University	
Pr HOBBAD	Lahoucine	Advanced Shrinkage Estimation for Symmetric Models	ENSA, Marrakech, Cadi Ayyad University	M. Alahiane (ENCG, El Jadida) and I. Ouassou (ENSA, Marrakech)
EL AAFANI	Mohamed Amine	Detecting Structural Change in Autoregressive Model Using Likelihood Ratio Test	FS, El Jadida, Chouaib Doukkali University	E. Elharfaoui (FS, El Jadida)
NEJJAM	Sara	Rupture detection methods	FS, El Jadida, Chouaib Doukkali University	E. Elharfaoui (FS, El Jadida)
ARROUCH	Mohamed Salah Eddine	A Generalized Test for Volatility Regime Shifts in Parametric Time Series	FS, El Jadida, Chouaib Doukkali University	E. Elharfaoui (FS, El Jadida) and J. Ngatchou-Wandji (Université de Lorraine)
LAGZINI	Abdelati	Estimating RO in Partially Observed Stochastic SIR Models: Bayesian and Classical Approaches	FST, Béni-Mellal Sultan Moulay Slimane University	H. El Maroufy and A. Merbouha (FST, Béni-Mellal)
Pr EL HARFAOUI	Echarif	Testing for multiple change-points for absolutely regular observations	FS, El Jadida, Chouaib Doukkali University	J. Ngatchou-Wandji (IECL, Université de Lorraine) and M. Harel (INSPE, Université de Limoges)

Pr EL OMARI	Mohamed	Parameter estimation of a tempered fractional Brownian motion with drift	FP, Sidi Bennour, Chouaib Doukkali University	
OUALAID	Abdelkarim	A mild approach to spatial discretization for backward stochastic differential equations in infinite dimensions	FSS, Marrakech, Cadi Ayyad University	H. Abidi (ESB, Tunis, Tunisia), Y. Ouknine (FSS, Marrakech) and R. Pettersson (Linnaeus University, Växjö, Sweden)
GOUTI	Hassan	Portfolio Optimization: Dynamic Programming Methods	FS, El Jadida, Chouaib Doukkali University	A. Gabih (FS, El Jadida)
SOULAIMANI	Sara	Analyse stochastique et stabilité globale d'un modèle de consommation de drogues avec étude de sensibilité statistique	ENSA, El Jadida, Chouaib Doukkali University	A. Kaddar (ENSA, El Jadida)
BENCHEIKH	Khadija	Heroin Use Dynamics with Relapse: A Stochastic Model with General Nonlinear Incidence	FS, El Jadida, Chouaib Doukkali University	Z. Bahraoui, Z. El Rhoubari (EST, Sidi Bennour) and R. Taki (FJSES, Salé)
LAOUINA	Aziza	Stationary Distribution and Extinction in a Stochastic Model of Multidrug-Resistant Tuberculosis	FS, El Jadida, Chouaib Doukkali University	Z. Bahraoui, Z. El Rhoubari (EST, Sidi Bennour) and R. Taki (FJSES, Salé)
Pr Balatif	Omar	Optimal control a spatio-temporal problem	FS, El Jadida, Chouaib Doukkali University	

### Topics: Stochastic Modeling, Mathematical Modeling, Optimal control

Pr FARAH	Elmehdi	Markovian-Switched SIR Epidemic Model with Two Coexisting Strains	ENSEM, Casablanca, Hassan 2 University	
EL YAHYAOUI	Mohamed	Deterministic and Stochastic Analysis of an Integrated Economic Model: Growth and Indicators Simulation	FST, Mohammedia, Hassan 2 University	S. Amine (FST, Mohammedia)
OULDKHOUIA	Noureddine	From Robbins-Monro to Adam: Stochastic Approximation Methods in Deep Learning Optimization	FS, Ben M'sik, Casablanca Hassan 2 University	I. Elberrai and K. Adnaoui (FS, Ben M'sik, Casablanca)
OULDKHOUIA	Noureddine	Towards a Probabilistic Theory of Deep Learning	FS, Ben M'sik, Casablanca Hassan 2 University	I. Elberrai and K. Adnaoui (FS, Ben M'sik, Casablanca)
IGBIDA	Noureddine	Mathematization of Competency Mastery Processes: Study of Three Effort Configurations	CMERF, Casablanca-Settat, SP El Jadida	N. Elharrar, J. El Meskaoui, M. Moukhliiss, A. Kaddar (ENSA, El Jadida)
IGBIDA	Jaouad	Modeling Intervention Strategies for Information Campaigns and Exact Information vs. Error Containment in Dynamic Social Networks	CMERF, Casablanca-Settat, SP El Jadida	Y. El Gali, A. Kaddar, N. Elharrar, B. Ennasiri (ENSA, El Jadida)
EL FARRAGUI	Meryiem	Analytical Solution of Oscillatory Newtonian Plane Couette Flow and Stokes' Second Problem with Dynamic Wall Slip	FS, El Jadida, Chouaib Doukkali University	O. Souhar (FS, El Jadida) and G. Georgiou (University of Cyprus, Nicosia, Cyprus)
EL FADILY	Sanaa	Stability Analysis of a Delay Differential Model for Unemployment Dynamics	EMI, Rabat, Mohammed V University	A. Kaddar (ENSA, El Jadida)

ELKAF	Meriem	Fractional Mathematical Analysis of Tobacco Consumption	Royal Naval School, Casablanca, Hassan 2 University	
BELHDID	Salaheddine	A Mathematical Framework for Fake Rumor Dissemination	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
OUYADRI	Mourad	Mathematical Model of the Monkeypox Epidemic with Waning Immunity and Staged Infectiousness	FS, El Jadida, Chouaib Doukkali University	
BENHACHEM	Anas	The Impact of Pricing Strategies and Advertising on Market Shares of two competing firms: A Differential Game Model Approach in an Oligopoly	FS, Ben M'sik, Casablanca, Hassan 2 University	O. Balatif (FS, El Jadida)
SMOUNI	Imane	Spatiotemporal Stability Analysis of Soil-Borne Disease Dynamics in Tomato Plants	FP, Khouribga, Sultan Moulay Slimane University	M. Belam (ENSEM, Casablanca)
EL MESKAOUI	Jaouad	A Mathematical Modeling and Analytical Approach to Controlling the Spread of Errors and Learning in Social Networks	FS, Ben M'sik, Casablanca, Hassan 2 University	J. Igbida (CRMEF, El Jadida), I. Elberrai (FS, Ben M'sik, Casablanca), N. Elharrar (ENSA, El Jadida), M. Moukhliiss (ESEF, El Jadida)
JABER	Tawfik	Gradient Optimal Control of an Infinite Dimensional Bilinear System Using Bounded Distributed Controls	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
BAROUDI	Mohamed	A coinfection Model of Bacterial Meningitis Diseases with Optimal Control	FP, Khouribga, Sultan Moulay Slimane University	A. Labzai and M. Belam (FS, Ben M'sik, Casablanca)
AARABATE	Oumaima	Optimal control analysis of an unemployment model with an online freelancing intervention	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
MOUJID	Salma	Stability Analysis of an Unemployment Model with Nonlinear Job Creation Dynamics	ENSA, El Jadida, Chouaib Doukkali University	M. El Aallaoui (FSJES, Ain Sebaa, Casablanca) and A. Kaddar (ENSA, El Jadida)
ZOUHRI	Samira	Stochastic Optimal Control of Epidemic Transmission	FS, Ben M'sik, Casablanca, Hassan 2 University	M. El Baroudi, H. Laarbi, M. Rachik (FS, Ben M'sik, Casablanca) and A. Abta (FP, Safi)
EL BAROUDI	Mohcine	Optimal Control of Epidemics with Time Delay	FS, Ben M'sik, Casablanca, Hassan 2 University	H. Laarbi, S. Zouhri, M. Rachik (FS, Ben M'sik, Casablanca) and A. Abta (FP, Safi)
ESSAMY	Amina	Optimal Control Strategies for Blood Glucose Regulation in Type 1 Diabetes	FS, Ben M'sik, Casablanca, Hassan 2 University	I. Dehaj, H. Ferjouchia, M. Rachik (FS, Ben M'sik, Casablanca)
Pr KADDAR	Abdelilah	Economic Modeling Using Mixed Differential Equations to Anticipate the Unexpected	ENSA, El Jadida, Chouaib Doukkali University	S. Soulaïmani (ENSA, El Jadida)
KHASSAL	Sofiane	Mathematical Modeling, Analysis, and Optimal Control of the cochineal insect impact on cacti plants	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)

OUBOUSKOUR	Khadija	Cost-Effectiveness Analysis of Control Strategies for Paratyphoid Transmission	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
LARRACHEE	Kawtar	Contribution of vaccination coverage in the fight against epidemics: A Mathematical vaccino- temporal SEIRS model	FS, Ben M'sik, Casablanca, Hassan 2 University	M. Karim, H. Ferjouchia, M. Rachik (FS, Ben M'sik, Casablanca)
MOUMINE	El Mehdi	Modeling and Analysis of a Fractional order Spatio-temporal SEIR Model: Stability and Prediction	FS, Ben M'sik, Casablanca, Hassan 2 University	
OUBOUSKOUR	Khadija	Optimal Control of Cutaneous Leishmaniasis Dynamics and Reservoir Limitation	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
HASSOUNI	Habib	Mathematical Analysis and Control of Merchant Behavior in E-Commerce Systems	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)
BALLAHI	Jamal	Modeling Foot-and-Mouth Disease under Active Immigration Context	FS, El Jadida, Chouaib Doukkali University	O. Balatif (FS, El Jadida)

### Topics: Mathematical methods in Dynamical systems

Pr IGHACHANE	Mohamed Amine	Jensen's Inequality for $M_\psi$ -Convex Functions and Operator Versions with Applications	ESEF, El Jadida, Chouaib Doukkali University	
EL BILALI	Mohammed	On the properties of the gDMP inverse of an operator	FS, El Jadida, Chouaib Doukkali University	M. Mouçouf (FS, El Jadida)
ELHYAT	Abdellah	Titchmarsh's theorem for the quaternion linear canonical transform	FS, El Jadida, Chouaib Doukkali University	A. Serhir (FS, El Jadida) and A. Achak (ESEF, El Jadida)
CHABLAOUI	Said	The quantum Adams-Bashforth-Moulton fractional method for solving quantum fractional differential equations	ENSA, El Jadida, Chouaib Doukkali University	L. Sadek (ENSA, AlHoceima) and E. Sadek (FS, El Jadida)
MOUNIR	Ayoub	On Skew Generalized Quasi-Cyclic Codes	FS, El Jadida, Chouaib Doukkali University	A. Haily and M. El Badry (FS, El Jadida)
KADYRI	Abbes	Adaptive Spline Interpolation Approach for Global MPPT in Photovoltaic Systems	FS, El Jadida, Chouaib Doukkali University	K. Kandoussi (ENSA, El Jadida) and O. Souhar (FS, El Jadida)
MABROUK	M'hamed	On the relations between different dual bilevel problems	FS, El Jadida, Chouaib Doukkali University	M. Laghdir, O. Balatif (FS, El Jadida)
ED-DAHDAH	Ahmed	Approximate strong and weak subdifferentials of the difference of two set-valued mapping	FS, El Jadida, Chouaib Doukkali University	M. Laghdir, O. Balatif (FS, El Jadida)

## Abstracts

Functional Projection Pursuit Regression within a Generalized Partially Linear Single-Index Framework

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**Abstract.** We introduced a functional approach to approximate the nonparametric function in the case of multivariate predictors, the single-index coefficient, the nonlinear regression function in the case of functional predictors and a scalar response. Following the Fisher-scoring algorithm and the principle of projection pursuit regression, we derive an additive decomposition that exploits the most predictive direction, the most predictive additive component of the functional predictor variable and the single-index component to explain the scalar response. On the one hand, this approach allows us to avoid the well-known problem of the curse of dimensionality in the nonparametric case with the notion of single index and the projection pursuit regression in the functional case, on the other hand, it can be used as an exploratory tool for the analysis of a multivariate and functional random variable belonging to a separable Hilbert space  $H$ . The terms of this decomposition are estimated with an iterative Fisher scoring procedure that uses the Quasi-Likelihood function and an approximation of the nonparametric function by normalized B-splines. The good behavior of our procedure is illustrated from a theoretical and practical point of view. Asymptotic results indicate that the nonparametric function, the single index coefficient and the terms of the additive decomposition can be estimated without suffering from curse of dimensionality, while some applications to real and simulated data show the high predictive performance of our method.

**Keyword.** Additive decomposition, Asymptotic normality, Fisher scoring algorithm, Functional data analysis (FDA), Polynomial splines, Predictive directions, Projection pursuit regression, Quasi-likelihood, Single-index model.

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Estimation in nonparametric functional-on-functional models with surrogate responses

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**Abstract.** We construct an estimator for the regression operator of a functional response variable using surrogate data, given a functional random variable. The almost complete uniform convergence rate of the estimator is then established. Finally, to demonstrate the predictive utility and superiority of the estimator when dealing with incomplete data, we apply the methodology to both simulated data and meteorological data.

**Keywords.** Almost complete convergence, Entropy, Functional data analysis, Kernel estimator, Regression operator, Semi-metric space, Surrogate response.

**Introduction.** Functional Data Analysis (FDA) has become a central field in modern statistics due to the increasing availability of complex data such as curves, images, and surfaces (Ferraty and Vieu, 2006; Goia and Vieu, 2016). Nonparametric regression is widely used to study the relationship between variables, with many results for scalar responses, but much fewer for functional responses (Ferraty et al., 2012; Cuevas, 2014). In practice, response functions are often incomplete, making estimation difficult. A common solution is to rely on surrogate data, which serve as substitutes when exact measurements of the response are unavailable (Wittes et al., 1989; Carroll and Wand, 1991). This idea, well studied in finite-dimensional settings, has recently been extended to FDA (Ibrahim et al., 2020). In this work, we propose an estimator of the regression operator for functional responses with surrogate variables and establish its almost complete uniform convergence. Finally, the efficiency of our method is illustrated with simulated data and a real meteorological application.

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Dimension Reduction and Variable Selection for High-Dimensional Right-Censored Data

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**Abstract.** Traditional statistical methods often struggle with the computational demands of analyzing massive datasets. We develop a new method, CSIR-WLS, which combines Sliced Inverse Regression (SIR) with Weighted Leverage Scores to simultaneously perform dimension reduction and variable selection for right-censored data. Based on extensive simulations and real-data applications, CSIR-WLS demonstrates superior or comparable performance in identifying key predictors across various correlation structures, making it a powerful and computationally efficient tool for high-dimensional data analysis.

**Keyword.** Survival analysis, censored data, Sliced Inverse Regression, reduction dimension, variable selection.

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High-Dimensional Sparse Clustering for Bank Marketing Data

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**Abstract.** This study introduces a new approach for clustering high-dimensional datasets that contain a mix of variable types. Our framework combines sparse clustering techniques with a quantile-based encoding of ordinal variables, ensuring that the specific characteristics of continuous, nominal, and ordinal data are properly respected. A key feature of the method is the use of Azzalini quantile-based scoring, which transforms ordinal variables into continuous scores while preserving their natural order and avoiding the loss of information that often occurs with traditional encodings. We also refine standard distance measures and adapt the Davies-Bouldin Index (DBI) to better capture the structure of mixed-type data. Through extensive simulations and a real-world study on bank marketing data, we show that this approach leads to clearer, more compact clusters, stronger separation between groups, and more effective variable selection. Overall, our method provides a robust and scalable solution for uncovering structure in large and complex datasets, with broad applicability across domains.

**Keywords.** Davies-Bouldin Index (DBI), Unsupervised Learning, Variable Selection, Ordinal data.

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Detecting error correlation in a periodic multiple regression model

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**Abstract.** This presentation proposes pseudo-Gaussian test for detecting error correlation in a periodic multiple regression model for unbalanced panel data. Simulation results demonstrate the validity and the power of the proposed pseudo-Gaussian test under both symmetric and asymmetric densities, as well as their superiority compared to the tests proposed by Baltagi et al. (2003).

**Keywords.** error correlation, periodic multiple regression model, pseudo-Gaussian test, PerRegMod package.

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Non parametric estimation by B-spline approach for the Gaussian Vasicek-type processes with random effects

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Abstract. This paper studies a linear stochastic differential equation whose solution is a Gaussian Vasicek-type process. A key feature of this model is that the drift term depends on an unobserved random effect, and the process is driven by a Gaussian process with specific properties. Our main contribution is the construction of a B-spline estimator for the unknown probability density of this random effect. We conduct a rigorous theoretical analysis of the estimator, establishing its asymptotic normality and studying its associated risks. The practical performance and validity of our theoretical results are confirmed through a comprehensive simulation study.

keywords: Non parametric estimation; Gaussian Vasicek-type process; B-spline.

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Mathematical Foundations of Vector Autoregressive Modeling and an Application to  
Macroeconomic Data

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**Abstract.** The objective of this paper is to present the main mathematical and statistical foundations of Vector Autoregressive (VAR) modeling. First, we will outline the mathematical formulation of the VAR model and the conditions for its stability. Next, we will present the main statistical tests for causality and robustness of such a model. Second, this paper will present an application of the aforementioned model to macroeconomic data using the EVIEWS software. Specifically, this application examines the effects of inflation, money supply, government spending, and the exchange rate on economic growth in the Moroccan case. It will also discuss the interpretation of the results from both a statistical and an economic perspective. These interpretations are based on two complementary analyses: impulse response analysis and forecast error variance decomposition.

**Keywords.** VAR modeling, Impulse Response Functions, Variance Decomposition.

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Advanced Shrinkage Estimation for Symmetric Models

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Abstract. This talk addresses the estimation of a location parameter for a d-variate random vector X with a spherically symmetric distribution under quadratic loss. We investigate three distinct shrinkage estimation methodologies aimed at developing estimators that dominate the standard estimator X. First, we build upon the work of Brandwein and Strawderman (1991) regarding the dominance conditions for estimators of the form  $X + ag(X)$ . We successfully remove one of their key technical conditions—specifically, the requirement that  $E_{-\theta}[R^2 h(V)]$  is nonincreasing in R. In doing so, we establish a new bound for the shrinkage coefficient a, which can offer improvement over the original bound:

$$0 < a < \frac{\mu_1}{d^2 \mu_{-1}} \left[ 1 - \frac{(d-1)\mu_1}{d\mu_{-1} - \mu_2} \right]$$

where  $\mu_i = E_0(\|X\|^i)$  for  $i = 1, -1, 2$ . This framework is further generalized to accommodate concave loss functions and cases where the scale parameter is unknown and a residual vector is present. Second, we consider a subclass of distributions inclusive of spherically symmetric ones to extend the findings of Evans and Stark (1996). Utilizing elementary stochastic analysis, we generalize a fundamental integration-by-parts lemma originally developed by Stein for the Gaussian case. This generalization allows us to derive more comprehensive conditions under which estimators of the form  $X + g(X)$  dominate X under quadratic loss. These results yield a broader class of improved estimators and are applicable in the presence of both known and unknown scale parameters.

Keywords. spherical symmetry, quadratic loss, James–Stein estimation, stochastic analysis, Brownian motion.

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Detecting Structural Change in Autoregressive Model Using Likelihood Ratio Test

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**Abstract.** This communication addresses the problem of detecting a single structural change in autoregressive model of order  $p$ . Following the discussion of parameter estimation, attention is directed to a robust change-point testing framework based on the Likelihood Ratio Test. The asymptotic distributions of the test statistics under the null hypothesis of structural stability are derived, and methodological approaches for accurately identifying the location of the change-point are examined. Practical procedures for localizing the change-point are assessed through extensive simulations, which confirm the convergence properties and reliability of the method across a wide range of scenarios. The approach is further validated on real-world data characterized by volatility shifts, and its performance is compared against recent algorithms from the literature.

**Keywords.** Autoregressive processes, Change-point detection, Statistical hypothesis test, Likelihood ratio, Simulations, Real-world data.

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A Generalized Test for Volatility Regime Shifts in Parametric Time Series

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**Abstract.** This paper proposes a novel testing procedure for detecting a single structural break in the volatility of parametric dependent time series. Unlike existing methods limited to proportional shifts, our approach tests for a change in the entire parameter vector of the conditional variance function. The methodology relies on a CUSUM-type statistic derived from a sequential marked empirical process of residuals. This semi-parametric framework avoids the need for a fully specified likelihood and applies to a broad class of stationary processes under strong mixing conditions. We establish asymptotic properties, including the consistency of both the test and the change-point estimator, and derive the null distribution of the test statistic. Monte Carlo experiments confirm accurate size control and strong power, while an application to S&P 500 daily returns reveals a clear volatility shift.

**Keywords.** Change-point detection; Time series analysis; Volatility shifts; Empirical processes; Strong mixing.

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Estimating  $R_0$  in Partially Observed Stochastic SIR Models: Bayesian and Classical Approaches

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**Abstract.** Estimating the basic reproduction number  $R_0$  is a central challenge in stochastic epidemic models, especially when data are only partially observed. In the SIR framework, the recovered compartment  $R(t)$  is typically well recorded, while the infected compartment  $I(t)$  is often indirectly or sparsely observed. We develop a unified Bayesian and frequentist framework for estimating  $R_0$  in partially observed stochastic SIR models. Transition probabilities between epidemic states are modeled using the Skellam distribution, leading to a tractable likelihood formulation combined with conjugate Gamma priors for transmission and recovery rates. An MCMC algorithm with data augmentation is proposed to jointly sample the latent  $I(t)$  trajectory and the posterior distribution of  $R_0$ . Classical profile likelihood estimators are also derived for comparison, with theoretical guarantees on weak consistency and asymptotic normality. Numerical simulations illustrate the performance of the proposed methods, demonstrating accurate and robust inference of  $R_0$  under partial observation. The framework provides a flexible and effective approach for analyzing stochastic epidemic data.

**Keywords.** Bayesian inference, Stochastic SIR models, Basic reproduction number, Data augmentation, Profile likelihood.

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Testing for multiple change-points for absolutely regular observations

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**Abstract.** We study multiple change-point detection tests for absolutely regular observations. The asymptotic distributions of the test statistics under the null hypothesis and under the local alternatives are given explicitly and the tests are shown to be consistent. A simulation experiment is done for evaluating the performance of the tests in detecting multiple changes in the mean, variance and autocorrelation of some simple times series.

**Keywords.** U-statistics, change-point, weak invariance, weak dependence, Wiener process.

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Parameter Estimation for a Tempered Fractional Brownian Motion with Drift

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Abstract. Tempered fractional Brownian motions (tfBm's) have been used to model wind speed data summarized by the so-called Davenport spectrum. The latter is a modification of the classical Kolmogorov spectrum for the inertial range of turbulence at low frequencies. TfBm's have also been used to describe geophysical flow data and in finance. The present work deals with the parametric estimation of the continuous-time model

$$X_t = \mu t + \sigma B_{H_\lambda}(t), \quad \forall t \geq 0$$

where  $B_{H_\lambda}$  is a tempered fractional Brownian motion. Based on discrete observations of  $X$  and using the method of moments with the ergodic theorem, the estimators of  $\mu$ ,  $\sigma^2$ ,  $H$ , and  $\lambda$  are formulated, and shown to be strongly consistent and asymptotically normal.

Keywords. Tempered fractional Brownian motion, Method of moments, Ergodic theorem, Consistency, Asymptotic normality.

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A Mild Approach to Spatial Discretisation for Backward Stochastic Differential Equations in Infinite Dimensions

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**Abstract.** In this paper, convergence for spatially semi-discrete approximation of backward stochastic differential equations (BSDEs) driven by a cylindrical Q-Wiener process taking values in a Hilbert space is considered. Both the solution and its spatial approximation are formulated in mild forms within the framework of the original paper [7]. The main result is a generalization of the work in [1], where the driver of the BSDE depends on the solution. Some crucial boundedness inequalities on extended martingale representations together with the Gronwall inequality are used. A delicate matter is to partition the time interval into subintervals and obtain convergence iteratively backward in time.

**Keywords.** Backward stochastic differential equations, Hilbert spaces, Extended martingale representation theorem.

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Stochastic Analysis and Global Stability of a Drug Use Model with Statistical Sensitivity

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**Abstract.** Our work examines the stochastic stability and global dynamics of a mathematical model of drug use. The model divides the population into five compartments current drug users, temporarily abstinent drug users, permanently abstinent drug users, and drug users in rehabilitation. Using Brownian motion, deterministic equations are extended to incorporate stochastic perturbations, capturing real-life uncertainties in drug use within compartments. An analysis of Lyapunov functions is used to determine the global stability of the model. By introducing stochastic elements into the model, we can examine its stability under random perturbations. A global sensitivity analysis, including PRCC results, is conducted to confirm the robustness of the model. Stable drug-free and drug-present equilibria can be maintained in both deterministic and stochastic environments. Numerical simulations illustrate the impact of various parameters on population dynamics and rehabilitation program effectiveness.

**Keywords.** Mathematical drug model, Global equilibrium stability, Lyapunov function analysis, Random perturbations, Stochastic stability analysis, Sensitivity analysis

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Heroin Use Dynamics with Relapse: A Stochastic Model with General Nonlinear Incidence

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**Abstract.** In this study, we adopt the assumption that heroin use can be modeled similarly to the spread of an infectious disease. We develop a heroin epidemic stochastic model with a nonlinear incidence rate. We first demonstrate that the system admits a unique global positive solution. Next, we derive criteria for the extinction and persistence of heroin users. Furthermore, we establish the existence of a unique ergodic stationary distribution for the system. Finally, numerical simulations are provided to support and illustrate the theoretical findings.

**Keywords.** Stochastic epidemic model, Extinction and permanence in mean, Stationary distribution.

2020 Mathematics Subject Classification: 93B70, 91G30.

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Portfolio Optimization: Dynamic Programming Methods

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**Abstract.** This paper investigates optimal trading strategies in a financial market in which stock returns depend on a hidden Gaussian mean-reverting drift process. Investors obtain information on that drift by observing stock returns. Moreover, expert opinions in the form of signals about the current state of the drift arriving at fixed and known dates are included in the analysis. Drift estimates are based on Kalman filter techniques. They are used to transform a power-utility maximization problem under partial information into an optimization problem under full information where the state variable is the filter of the drift. The dynamic programming equation for this problem is studied and closed-form solutions for the value function and the optimal trading strategy of an investor are derived. They allow quantification of the monetary value of information delivered by expert opinions. We illustrate our theoretical findings through extensive numerical experiments. This talk is based on work by Abdelali Gabih, Hakam Kondakji, and Ralf Wunderlich [3].

**Keywords.** Power utility maximization, Partial information, Stochastic optimal control, Kalman-Bucy filter, Expert opinions, Black-Litterman model.

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Stationary Distribution and Extinction in a Stochastic Model of Multidrug-Resistant Tuberculosis

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**Abstract.** The principal focus of this work is the study of a stochastic model for multidrug-resistant tuberculosis (MDR-TB). First, we investigate the existence of a unique positive solution to the proposed system. Next, by constructing a suitable random Lyapunov function, we establish sufficient conditions for the existence of a stationary distribution. In addition, conditions for extinction are also provided.

**Keywords.** Stochastic epidemic model, Multidrug-resistant tuberculosis, Stationary distribution, Extinction conditions.

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Markovian-Switched SIR Epidemic Model with Two Coexisting Strains

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Abstract. In this paper, we investigate a stochastic epidemic model involving two interacting epidemics, incorporating both white noise and telegraph noise represented through Markovian switching. We derive sufficient conditions for disease extinction and persistence. Finally, numerical simulations are provided to illustrate and validate the theoretical findings.

Keyword. Stochastic epidemic model, Markovian switching, Regime switching dynamics.

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Deterministic and Stochastic Analysis of an Integrated Economic Model: Growth and Indicators Simulation

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**Abstract.** This work develops a mathematical framework for analyzing economic growth by integrating deterministic and stochastic approaches. The deterministic part describes the evolution of key macroeconomic variables such as capital stock, consumption, investment, and interest rates, while the stochastic extension accounts for random fluctuations and uncertainty. Stability analysis is conducted to explore the conditions under which equilibrium is maintained, and stochastic processes are used to capture variability in growth dynamics. Numerical simulations and parameter estimation highlight the influence of savings and related factors on system behavior. The proposed framework contributes to the mathematical modeling of socioeconomic phenomena by linking deterministic structures with stochastic processes, offering insights relevant to statistics, probability, and dynamical systems.

**Keywords.** Integrated Economic Model · Growth · Local stability · Brownian motion · Calibration · Numerical simulations

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Towards a Probabilistic Theory of Deep Learning

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**Abstract.** We propose a unified perspective linking the training dynamics of deep neural networks with tools from stochastic process theory. By treating parameter updates (SGD, minibatch, adaptive algorithms) as discretization schemes of stochastic differential equations (SDEs), we obtain a framework for analyzing convergence, stability, and generalization from probabilistic objects (invariant measures, Fokker–Planck equation, metastable phenomena). This perspective sheds light on several open questions: why sampling noise favors flat minima, how the type and intensity of noise modulate generalization capacity, and within what limits uncertainty propagation can be controlled. We present (i) a mathematical formulation linking SGD/SGLD to the corresponding SDEs, (ii) analytical results on the limit law and minima escape dynamics for simple models, and (iii) comparative numerical experiments (SGD vs. SGLD vs.  $\alpha$ -stable noise, wide vs. narrow networks) showing predictable statistical signatures. Finally, we discuss connections with mean-field approaches, kernels (NTK), and PAC-Bayes frameworks, and propose a roadmap to extend the theory to modern architectures (transformers, residual networks). This probabilistic theory aims to provide analytical and practical tools to design more robust and better controlled deep learning algorithms.

**Keywords.** Probabilistic learning theory, Bayesian deep learning, Stochastic differential equations.

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From Robbins–Monro to Adam: Stochastic Approximation Methods in Deep Learning Optimization

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**Abstract.** The optimization of deep neural networks has its roots in classical stochastic approximation methods. The Robbins–Monro algorithm (1951) introduced recursive updates designed to solve equations with noisy observations. Similarly, the Kiefer–Wolfowitz approach (1952) proposed finite-difference estimators of the gradient, allowing stochastic optimization even when gradients are unavailable. Modern Stochastic Gradient Descent (SGD) and its variants Momentum, RMSProp, and Adam can be interpreted as extensions of these pioneering methods. From a stochastic process viewpoint, SGD admits a continuous-time approximation revealing strong connections with stochastic differential equations (SDEs) and Langevin dynamics. This perspective highlights that modern optimization in deep learning is not only algorithmically efficient but also theoretically grounded in stochastic approximation theory. Our contribution emphasizes this link, proposing a unified framework to reinterpret adaptive gradient methods as stochastic approximation schemes with memory and preconditioning. This approach opens new directions for the design of hybrid algorithms suitable for high-dimensional machine learning problems where gradients are costly or noisy.

**Keywords.** Stochastic approximation, Robbins–Monro, Kiefer–Wolfowitz, Stochastic Gradient Descent (SGD), Adam, RMSProp, Optimization, Deep Learning.

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Modeling Intervention Strategies for Information Campaigns and Exact Information vs. Error Containment in Dynamic Social Networks

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**Abstract.** This paper introduces a novel framework for modeling intervention strategies in social networks, focusing on the concurrent challenges of promoting beneficial exact information and containing a competing error. We develop a compartmental model that segments the population into distinct states: Susceptible, Exact-Information-Aware, Error-Believer, and Recovered. The model captures the competitive dynamics of information diffusion, where the spread of the exact information and the error are mutually inhibitory. We establish the well-posedness of the model, including the existence, uniqueness, and positivity of solutions. The analysis of the system's equilibria reveals critical thresholds for the effective dissemination of the exact information. To design an optimal control policy, we formulate an objective functional that seeks to maximize the reach of the exact information while minimizing both the prevalence of the error and the cost of intervention efforts over a finite time horizon. Using Pontryagin's Maximum Principle, we characterize the optimal control strategy. Numerical simulations across diverse network topologies and parameter settings demonstrate that a time-varying strategy, which aggressively promotes the exact information initially and adapts based on error prevalence, outperforms static interventions. Our results provide actionable insights for public health communication and error mitigation.

**Keywords.** Social Networks, Optimal Control, Exact Information, Error Modeling, Compartmental Models, Pontryagin's Maximum Principle.

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Analytical Solution of Oscillatory Newtonian Plane Couette Flow and Stokes' Second Problem with Dynamic Wall Slip

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**Abstract.** We solve analytically two unsteady Newtonian flows, i.e., the plane Couette flow and Stokes' second problem, initiated by the oscillatory motion of a wall in the presence of dynamic wall slip. The velocity is expressed as the sum of a transient and a periodic component and explicit expressions are derived for both components using the Laplace transform method. The effects of the slip relaxation parameter on both velocity components and the corresponding shear stresses are investigated and comparisons with available results in the literature are made. In general, wall slip delays the evolution to periodic flow and this effect is further enhanced by increasing the slip relaxation parameter. The contribution of the Navier slip parameter to the periodic component of the solution comes with a negative phase difference for the velocity component and is in phase with the shear stress, while the contributions of the dynamic slip parameter come with positive phase differences for both the velocity and the shear stress.

**Keywords.** Analytical Method, Dynamic Wall Slip, Oscillatory Couette Flow, Stokes' Second Problem.

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Stability Analysis of a Delay Differential Model for Unemployment Dynamics

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**Abstract.** In this work, we study the global dynamics of a delay differential model describing unemployment evolution by incorporating key labor market variables such as the unemployed population, employment, and job vacancies. The model accounts for time delays associated with training or preparation before employment. Using Lyapunov functionals and LaSalle's invariance principle, we establish sufficient conditions for the global asymptotic stability of both the unemployment free and positive equilibria. The analysis reveals that the long-term behavior of the system depends critically on the basic reproduction number  $R_0$ . Numerical simulations are presented to illustrate the theoretical findings and examine the effect of various economic parameters.

**Keywords.** Unemployment model; Delay differential equations; Global stability; Lyapunov functionals; LaSalle's invariance principle; Basic reproduction number.

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Fractional Mathematical Analysis of Tobacco Consumption

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**Abstract.** This paper presents a mathematical analysis of fractional dynamics in tobacco consumption, aiming to provide insights into the complex dynamics of smoking behavior and its implications for public health. Fractional calculus offers a powerful tool for modeling systems with memory, long-range dependence, and non-local interactions, making it well-suited for capturing the multifaceted nature of tobacco use dynamics. By employing fractional differential equations and associated boundary conditions, we investigate the dynamics of smoking prevalence, cessation rates, and public health outcomes. Through mathematical analysis and numerical simulations, we explore the impact of various factors on tobacco consumption patterns and assess the effectiveness of intervention strategies. Our findings underscore the importance of considering fractional dynamics in understanding and addressing the challenges posed by tobacco consumption, offering valuable insights for policymakers.

**Keywords.** Tobacco Consumption, Fractional Differential Equations, Dynamics Stability, Numerical Analysis.

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A Mathematical Framework for Fake Rumor Dissemination

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**Abstract.** This paper delves into the pervasive issue of misinformation and disinformation propagation within social networks. Employing a compartmental model, inspired by epidemiological modeling, we characterize the dynamics of information diffusion as it spreads through different segments of a population. The model incorporates distinct compartments representing individuals susceptible to misinformation, actively spreading it, and those who have developed immunity to its influence. To mitigate the detrimental impact of misinformation, we introduce optimal control strategies that dynamically manipulate key parameters influencing the spread of false information. Leveraging control theory, we formulate an optimization problem to minimize the prevalence of misinformation while considering resource constraints and ethical considerations. Our findings highlight the effectiveness of targeted interventions in curtailing the dissemination of misinformation. The proposed compartmental model, coupled with optimal control strategies, provides valuable insights for policymakers and social media platforms seeking evidence-based approaches to counteract the harmful effects of false information in contemporary communication ecosystem. [1] [2] [3].

**Keywords.** Optimal control, rumors, compartment models, social network.

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Mathematical Model of the Monkeypox Epidemic with Waning Immunity and Staged Infectiousness

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**Abstract.** This research constructs a compartmental epidemic model for analyzing monkeypox transmission dynamics specifically regarding waning immunity as well as staged infectiousness. The  $SEI_1I_2RW$  model adds two infectious compartments ( $I_1$  and  $I_2$ ) to capture differential transmission during progressive clinical stages and includes a waned immunity compartment ( $W$ ) for loss of protective immunity. We prove the model's well posedness and derive the basic reproduction number  $R_0$  and equilibrium behavior alongside some nonlinear dynamical analysis.  $R_0 < 1$  shows the disease-free equilibrium local asymptotic stable while endemic equilibrium emerges and remains stable when  $R_0 > 1$ . In the context of the analysis, the decay immunity loss rate  $\mu$  is identified as the most critical for long term epidemic behavior. Also, the value of transmission rate of the disease between stages of the disease  $\beta$  more heavily influences outbreak size than is deemed appropriate, highlighting the importance of timed, targeted, and staged intervention. The results presented form a basis for assessing the impact of different public health interventions, in particular, the timing of booster vaccination doses and targeted isolation of confirmed cases. The model's structure serves as a versatile framework for future extensions incorporating additional epidemiological complexities.

**Keywords.** Monkeypox, mathematical epidemiology, waning immunity, infectious disease modeling, stability analysis, reproduction number.

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The Impact of Pricing Strategies and Advertising on Market Shares of two competing firms: A Differential Game Model Approach in an Oligopoly

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**Abstract.** In this work we investigate the dynamic interaction between pricing and advertising strategies of two competing firms in an oligopolistic market through the lens of differential game theory. Each firm seeks to maximize its intertemporal profit function while influencing the evolution of market shares. The state variable is defined as the relative market share, which evolves according to a nonlinear dynamic system driven by pricing and advertising efforts. Both firms select their control variables that are price and advertising intensities while anticipating the strategic behavior of their rival. By applying the Pontryagin maximum principle, we derive the open-loop Nash equilibrium and analyze the joint dynamics of states and costates. The resulting equilibrium trajectories illustrate the trade-off between aggressive pricing policies and the long-term benefits of advertising, as well as the impact of competition intensity and advertising effectiveness on market share dynamics. This framework not only provides theoretical insights into optimal competitive behavior in oligopolistic industries but also offers a mathematical foundation for managerial decision-making in dynamic markets.

**Keywords.** Differential Games, Oligopoly, Pricing Strategies, Advertising, Market Share Dynamics, Dynamic Competition, Open-Loop Nash Equilibrium.

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A Mathematical Modeling and Analytical Approach to Controlling the Spread of Errors and Learning in Social Networks.

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**Abstract.** This study develops a mathematical model to understand how correct knowledge and errors spread in educational social networks. The model shows how students can be influenced either by accurate information or by mistakes, and how these two forms of knowledge interact. We analyze the stability of different situations: when only correct learning exists, when only errors spread, and when both coexist. To reduce errors, we apply control strategies based on Pontryagin's Maximum Principle. These strategies guide students toward reliable sources while limiting exposure to misinformation. Numerical simulations confirm that combining learning promotion with error reduction is the most effective and cost-efficient approach.

**Keywords.** Information diffusion; Online social networks; Mathematical modeling; Information control; Disinformation

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Stability Analysis and Optimal Control of a Mathematical Model for Cancer Drug Resistance

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**Abstract.** We present a novel mathematical model governed by a bilinear system of ordinary differential equations, designed to capture the dynamic interactions between two groups of cancer subpopulations under combined therapies. This model is distinctive in that it integrates two key approaches in cancer modeling: drug resistance dynamics and cell cycle-specific progression, allowing us to study both how cells acquire resistance and how they progress through different stages of growth. The model incorporates phenotypic transitions, proliferation across cell cycle phases, and therapeutic responses. Through a detailed stability analysis, we derive critical threshold values that determine the persistence or elimination of the tumor population and rigorously establish the existence and nature of the system's equilibria. To investigate treatment optimization, we formulate and analyze an optimal control problem aimed at minimizing the resistant cell population while limiting treatment intensity and preserving the sensitive cell line. Using Pontryagin's Maximum Principle, we derive the necessary conditions for optimality and employ a numerical forward-backward sweep method to solve the resulting system. Numerical simulations implemented in MATLAB illustrate how specific treatment combinations and scheduling can significantly influence tumor progression and therapeutic outcomes.

**Keywords.** Bilinear systems, stability analysis, optimal control.

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A coinfection Model of Bacterial Meningitis Diseases with Optimal Control

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**Abstract.** Meningitis is a life-threatening infection of the meninges, with high global morbidity and mortality, particularly in the sub-Saharan "meningitis belt". The main bacterial agents include *Neisseria meningitidis*, *Streptococcus pneumoniae*, and *Hemophilus influenzae*, among others. Despite progress achieved through vaccination, surveillance, and case management, recurrent outbreaks remain a major public health concern. In this study, we develop and analyze an extended SEAIVHRQ compartmental model that incorporates realistic features such as voluntary quarantine (Q) and proactive vaccination (V). The model captures both medical and behavioral interventions, reflecting challenges faced in resource-limited settings. To improve epidemic control, three time-dependent control functions are introduced: reducing transmission from asymptomatic carriers, limiting contact and transmission from symptomatic individuals, and promoting awareness and treatment adherence. Using optimal control theory, we investigate the effectiveness of these interventions in minimizing infection levels, reducing mortality, and optimizing healthcare resources. Numerical simulations demonstrate that combining vaccination, isolation, quarantine, and awareness campaigns significantly mitigates disease burden. This work contributes to the design of cost-effective strategies aligned with the WHO "Defeating Meningitis by 2030" roadmap, and offers practical insights for policymakers in endemic regions.

**Keywords.** Control optimal, Meningitis, MATLAB.

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Optimal control analysis of an unemployment model with an online freelancing intervention

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Abstract. In the digital age, freelancing on the Internet has become a popular trend as an alternative to traditional employment and has an enormous impact. The purpose of this study is to investigate the relationship between unemployment and online freelancing. Thus, online freelancing can generate jobs in developing nations. So, we suggested a mathematical model that describes the dynamics of the unemployment problem with an online freelancing intervention. Based on the model, an optimal control problem is suggested and many sensible and appropriate control strategies are proposed. The optimal controls are characterized by Pontryagin's minimum principle, and an iterative method is employed to solve the optimality system. Finally, some numerical simulations are run using MATLAB to confirm the theoretical analysis.

Keywords. Mathematical modelling, unemployment model, online freelancing, optimal control theory.

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Stability Analysis of an Unemployment Model with Nonlinear Job Creation Dynamics

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**Abstract:** In this study, we develop a new mathematical framework for modeling unemployment that integrates nonlinear job creation and a matching mechanism to enhance the model's realism. We establish the existence and uniqueness of solutions, and examine both the local and global stability of the equilibrium points. Moreover, two optimal control strategies are incorporated into the system to mitigate the persistence of unemployment. Finally, a numerical scheme is implemented to validate and illustrate the theoretical results.

**Keywords.** Unemployment model, Nonlinear job creation, Stability analysis, Optimal control.

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Stochastic Optimal Control of Epidemic Transmission

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**Abstract.** The primary aim of this study is to evaluate the impact of mask usage and active screening/testing in controlling the spread of COVID-19 once transmission has already begun. Unlike many works that prioritize costly measures such as travel restrictions or vaccination campaigns [1], this article highlights the often-overlooked role of basic interventions. To this end, we formulate a stochastic SEIR model that accounts for four compartments: susceptible, exposed, infected, and recovered individuals [2]. Two control variables are incorporated to represent mask-wearing and active screening/testing. By applying an adapted version of Pontryagin's maximum principle designed for stochastic systems [3], we investigate the optimal strategies for implementing these measures. The optimality system is solved using the ForwardBackward Sweep Method (FBSM) in combination with a tailored Runge-Kutta scheme for stochastic differential equations. Our findings indicate that, even after the onset of community transmission, the combined application of mask use and active screening/testing substantially reduces the numbers of exposed and infected individuals, thus providing a cost-effective approach for resource-constrained settings [4].

**Keywords.** SEIR model, stochastic differential equation, stochastic optimal control, stochastic maximum principle, COVID-19.

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Optimal Control of Epidemics with Time Delays

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**Abstract.** This work investigates the impact of delayed preventive measures on the spread of COVID-19. We focus on the effectiveness of interventions such as mask-wearing, active screening and testing, and vaccination when their implementation is not immediate. To capture these effects, we propose a SEIR epidemic model with multiple time delays in both the state and control variables. The delayed optimal control problem is formulated and solved using Pontryagin's Maximum Principle, leading to necessary conditions for optimal strategies under state-control constraints. Numerical simulations highlight that, after a delay phase, immediate enforcement of mask use and vaccination campaigns is essential to reduce infection levels, while active screening and testing further accelerate epidemic control. Our findings provide theoretical insights into the timing of public health interventions and offer guidance for mitigating the adverse consequences of delayed responses during epidemic outbreaks.

**Keywords.** Epidemic modeling, SEIR model, Optimal control, Time-delays, State and control variables.

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Optimal Control Strategies for Blood Glucose Regulation in Type 1 Diabetes

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**Abstract.** Type 1 diabetes (T1D) is a chronic condition characterized by the pancreas's inability to produce insulin, necessitating external regulation of blood glucose levels. This study investigates the application of “optimal control strategies” to a “physiological model of T1D”, specifically an extended Bergman minimal model incorporating a meal compartment  $m(t)$  and a control term  $u(t)$ . The objective is to minimize insulin administration while maintaining blood glucose within a safe range, addressing the trade-off between glycemic control and hypoglycemia risk. We propose a mathematical framework combining “Pontryagin's Maximum Principle” and “numerical optimization” to derive an optimal insulin infusion rate  $u^*(t)$ . The control problem is formulated with a cost functional that penalizes both hyperglycemia and excessive insulin use, subject to physiological constraints. The meal compartment  $m(t)$  is modeled as a dynamic input, allowing realistic simulation of postprandial glucose excursions.

Key contributions include:

1. Comparison of control architectures:
  - Feedback-linearization for real-time adjustments.
  - Model Predictive Control (MPC) for handling meal disturbances.
2. Sensitivity analysis of model parameters to optimize robustness.
3. In-silico validation using clinical data, demonstrating reduced glycemic variability compared to standard therapy.

Results indicate that optimal control significantly improves glucose regulation while minimizing insulin doses.

**Keywords.** Optimal control theory, Minimal model (Bergman model), Insulin-glucose dynamics, Biomedical control systems, Type 1 diabetes.

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Economic Modeling Using Mixed Differential Equations to Anticipate the Unexpected

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**Abstract.** Our work this article explores economic modeling using mixed differential equations, highlighting the dynamics of business cycles and their impact on economic stability and instability. Business cycles, characterized by phases of growth and recession, are influenced by internal and external factors. By incorporating lags and leads into the equations, we analyze the mechanisms underlying these cycles. We identify bifurcation points, where small variations in parameters can lead to significant changes in economic behavior. The numerical simulation method makes it possible to assess the effects of external shocks and predict transitions between stable and unstable states. The results show that mixed differential models provide a better understanding of complex economic behavior, making it possible to anticipate unexpected events and provide sound recommendations for decision-makers. This work highlights the importance of a dynamic approach to economic analysis to effectively navigate a constantly changing environment.

**Keywords.** Mixed differential equations, stability, instability, bifurcation, economic model, Fluctuations.

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Mathematical Modeling, Analysis, and Optimal Control of the cochineal insect impact on cacti plants

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**Abstract.** We propose a mathematical model, SIM, that describes the dynamics of cochineal insect spread among cacti and examines the effects of various control strategies. The model is analyzed for the existence and uniqueness of solutions, and we investigate the equilibrium points and stability of the system using both local and global stability analyses. By performing numerical simulations in MATLAB, we validate our theoretical findings. Furthermore, we propose an optimal control strategy to minimize the cochineal population in cacti fields. The optimal control problem is formulated using Pontryagin's maximum principle, and the corresponding optimality system is solved iteratively. Our study compares three control strategies: cutting and burning infected cacti, insecticide spraying, and a combined approach. The results demonstrate that the combined strategy is the most effective in reducing the cochineal population. This research provides valuable insights into managing cochineal infestations and offers practical recommendations for farmers to control the spread of these pests.

**Keywords.** Mathematical modeling, Stability analysis, Optimal control, Cochineal insect, Cacti plants.

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Cost-Effectiveness Analysis of Control Strategies for Paratyphoid Transmission

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**Abstract.** This paper presents the development of a sophisticated mathematical model aimed at optimizing the control of paratyphoid in Taiwan. The model incorporates key factors influencing transmission dynamics, including human interactions, exposure to contaminated sources, and the effectiveness of preventive measures. It is analyzed using optimal control theory and rigorous numerical simulations, allowing for a comprehensive assessment of various intervention strategies. The findings highlight that a combination of targeted sanitation measures and educational awareness campaigns constitutes the most effective approach to reducing disease spread. This work underscores the critical role of mathematical modeling as a decision-support tool for public health planning, providing data-driven insights to guide the prevention and control of paratyphoid.

**Keywords.** Pontryagin's maximum principle, Paratyphoid, Hamiltonian, Cost-effectiveness.

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Contribution of vaccination coverage in the fight against epidemics: A Mathematical vaccino-temporal SEIRS model

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**Abstract.** In this paper, we consider a SEIRS-type mathematical model describing the spread of an epidemic within a population. The model comprises four compartments: susceptible  $S(I, x)$ , exposed  $E(I, x)$ , infected  $I(I, x)$ , and recovered  $R(I, x)$ , where  $i$  denotes time and  $x$  the number of vaccine doses received by an individual. We show, through the minimization of a suitable cost functional, that effective awareness campaigns regarding vaccination can significantly reduce the number of infected individuals.

**Keywords.** SEIRS Model, Optimal Control, Dynamic Vaccination, Numerical Simulation

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Optimal Control of Cutaneous Leishmaniasis Dynamics and Reservoir Limitation

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**Abstract.** This study presents a SEIR-type mathematical model to investigate the transmission dynamics of cutaneous leishmaniasis. The model captures disease interactions among humans, sandflies, and animal reservoirs, highlighting the dual infection pathways of sandflies from both humans and animals. Using advanced mathematical and optimal control techniques, we evaluate various intervention strategies, including vector control, early diagnosis and treatment, public awareness, and monitoring of infected animal hosts. The results demonstrate that integrated control measures are significantly more effective than individual interventions. This model underscores the importance of coordinated human and animal health programs and provides a practical framework for decision-makers to optimize resource allocation in endemic regions. The proposed approach can also be adapted to other vector-borne diseases.

**Keywords.** Pontryagin's maximum principle, Leishmaniasis, Hamiltonian.

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Modeling Foot-and-Mouth Disease under Active Immigration Context

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**Abstract.** We study a compartmental SEIAR model of foot-and-mouth disease (FMD) that integrates infectious immigrant flows, both asymptomatic and symptomatic. The basic reproduction number  $R_0$  is derived using the next-generation matrix. The disease-free equilibrium is locally stable according to the Routh–Hurwitz criterion and globally stable (Castillo–Chavez approach) when  $R_0 < 1$ . A forward bifurcation occurs for  $R_0 > 1$ . Numerical simulations (Runge–Kutta 4th-order method, MATLAB) show that infectious migration accelerates the depletion of susceptible and alters the trajectories toward endemic equilibrium. These results highlight the need for coordinated policies involving vaccination, quarantine, and cross-border surveillance.

**Keywords.** Foot-and-mouth disease; Mathematical modeling; Active immigration; Stability;  $R_0$

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Jensen's Inequality for  $M_\psi$ -Convex Functions and Operator Versions with Applications

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Abstract. In this paper, we present a comprehensive generalization of the classical Jensen inequality through a continuous, strictly monotone function  $\psi$ , leading to the concept of  $M_\psi$ -convex functions. This framework extends the classical convexity setting and yields the integral inequality

$$f(\psi - 1(\int X\psi(g(x)) d\mu(x))) \leq \int X f(g(x)) d\mu(x),$$

which unifies several well-known convexity types such as arithmetic, geometric, harmonic, and power means under a single analytic scheme. We further establish operator versions of this inequality in complex Hilbert spaces, showing that for any self-adjoint bounded operator

$A \in B(H)$  and every unit vector  $x \in H$ ,

$$f(\psi - 1(\langle \psi(A)x, x \rangle)) \leq \langle f(A)x, x \rangle.$$

This generalized Jensen framework provides a unified and powerful tool for deriving refined bounds and Jensen-type inequalities in both scalar and operator contexts. Several special cases and refinements are also discussed, together with applications to operator norm inequalities involving weighted means such as geometric, harmonic, and power means.

Keywords.  $M_\psi$ -convexity, Jensen's inequality, operator inequalities, convex functions, operator means, Hilbert space operators, functional calculus.

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On the Properties of the gDMP Inverse of an Operator

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**Abstract.** This work explores the gDMP inverse associated with generalized Drazin invertible operators possessing a closed range. We provide several characterizations and investigate fundamental properties of the gDMP inverse. Furthermore, we establish its generalized Drazin invertibility and identify the conditions under which it coincides with the Moore–Penrose inverse. Key properties are derived, and applications to the solution of specific linear operator equations are discussed.

**Keywords.** Generalized Drazin inverse, Moore–Penrose inverse, gDMP inverse.

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Titchmarsh's Theorem for the Quaternion Linear Canonical Transform

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Abstract. In this presentation we will give some results associated with Dini–Lipschitz functions in  $L_r(\mathbb{R}^2, \mathbb{H})$ ,  $1 \leq r < 2$  where  $\mathbb{H}$  is the quaternion algebra, for quaternion linear canonical transform. In particular, we give a generalized Titchmarsh's theorem for measurable sets from complex domain to hypercomplex domain by using quaternion algebras, associated with the quaternion linear canonical transform.

Keywords. Quaternion linear canonical transform, Dini–Lipschitz class, Titchmarsh theorem.

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The quantum Adams-Bashforth-Moulton fractional method for solving quantum fractional differential equations

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Abstract. In this work, we present the quantum Adams-Bashforth-Moulton fractional method for solving the Caputo quantum fractional differential equation on the time scale set

$$Tq(cD_{\varpi} q \Xi(\Theta) = f(\Theta, \Xi(\Theta)), \Xi(k)(a) = \Xi(k) a, k = 0, 1, \dots, [\varpi] - 1,$$

with  $\varpi > 0$ ,  $0 < q < 1$  and the differential operator is the Caputo type quantum derivative. We study the stability of the solution, also give a detailed error analysis. Finally, numerical examples including linear and nonlinear are provided to illustrate the robustness of our method.

Keywords. q-fractional differential equation, Caputo q-derivative, q-Adams-Bashforth-Moulton fractional method, error analyses.

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On Skew Generalized Quasi-Cyclic Codes

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**Abstract.** In this work, we consider the finite non-chain ring  $R = Z_4 + uZ_4$  with  $u^2 = 1$ . We introduce a new family of codes, namely  $(\sigma, \delta)$ -skew generalized quasi-cyclic (GQC) codes over  $R$ , where  $\sigma$  is an automorphism of  $R$  and  $\delta$  is a  $\sigma$ -derivation of  $R$ . We investigate the structure of 1-generator  $(\sigma, \delta)$ -skew GQC codes and establish a sufficient condition ensuring that such codes are free. In addition, we provide a lower bound on the minimum distance of free 1-generator  $(\sigma, \delta)$ -skew GQC codes.

**Keywords.** Skew cyclic codes, Skew generalized quasi-cyclic codes, Skew polynomial rings.

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Adaptive Spline Interpolation Approach for Global MPPT in Photovoltaic Systems

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**Abstract.** We propose an adaptive spline interpolation method for global MPPT in photovoltaic systems under partial shading. By switching between linear and cubic splines and applying a two-stage optimization, the approach ensures accurate and efficient MPP detection. Simulations and semi-experimental results show superior accuracy and robustness compared to P&O and PSO, making it suitable for real-time PV applications.

**Keywords.** Photovoltaic Systems; Global Maximum Power Point Tracking (GMPPT); Adaptive Spline Interpolation; Perturb and Observe (P&O); Particle Swarm Optimization (PSO).

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Mathematization of Competency Mastery Processes: Study of Three Effort Configurations

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**Abstract.** This article presents a mathematical modeling approach to competency mastery dynamics using linear dynamical systems. Building on knowledge retention theories and skill acquisition frameworks, we develop a differential equation model that formalizes the evolution of competency levels over time. The model incorporates two fundamental parameters: learning rate ( $\alpha$ ) and forgetting rate ( $\beta$ ), operating under varying effort configurations. We simulate and analyze three distinct scenarios: zero effort (complete skill decay), moderate effort (stable competency maintenance), and intensive effort (expertise development). Our results demonstrate quantitatively how different effort strategies lead to fundamentally different competency trajectories and equilibrium states. The model provides a formal framework for optimizing training investments, designing maintenance strategies, and understanding the time dynamics of skill development. This mathematical approach bridges qualitative pedagogical theories with predictive quantitative analysis, offering valuable insights for educational design and human resource development.

**Keywords.** Skill Dynamics, Nonlinear Systems, Mathematical Modeling, Motivational Feedback, Learning, Dynamics Optimization.

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On the relations between different dual bilevel problems

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**Abstract.** For a bilevel programming problem with an extremal-value function we determine, by means of the conjugacy approach based on the perturbation theory, some dual problems to it. The relations between the optimal objective values of these duals are studied. Moreover, sufficient conditions are given in order to achieve equality between the optimal objective values of the duals and strong duality between the primal and the dual problems, respectively.

**Keywords.** Convex programming, Perturbation theory, Composed programming problems.

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Approximate strong and weak subdifferentials of the difference of two set-valued mappings

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**Abstract.** The aim of this paper is to establish necessary and sufficient approximate optimality condition for the difference of two set-valued mappings in a finite or infinite-dimensional preordered space, characterizing the approximate strong and weak efficient solutions. Our approach is based essentially on the calculus rule for the strong and weak approximate subdifferentials of the difference of two set-valued mappings, expressed in terms of the star difference.

**Keywords.** Approximate optimality conditions, DC vector set-valued optimization, Pareto  $\varepsilon$ -subdifferential.

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